The algebraic side of the N-body problem: reduction, singularities, relative equilibria

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Abstract

The course will start with a description of the reduction of symmetries in the N-body problem: in his fondamental "Essai sur le problème des trois corps" (Essay on the 3-body problem [L1]), Lagrange, well before Jacobi's *reduction of the node*, carried out the first complete reduction of symetries in this problem. Discovering the so-called *homographic motions* (Euler had treated only the colinear case), he showed that these motions necessarily take place in a fixed plane, a result which is simple only for the *relative equilibria*. In order to understand the true nature of this reduction – and of Lagrange's equations - one must consider the N-body problem in an euclidean space of dimension $2(N-1)$, the actual dimension of the ambient space appearing as a constraint, namely the angular momentum bivector's degeneracy. I shall describe the results obtained in [AC] (see also [C1]): a non homothetic homographic motion is always directed by a complex structure, which forces the space where the motion actually takes place to be even dimensional. Two cases are possible: either the configuration is *central* (that is a critical point of the potential among configurations with a given moment of inertia with respect to the center of mass) and periodic homographic motions with arbitrary eccentricity exist as soon as the configuration lives in an even dimensional space, or it is *balanced* (that is a critical point of the potential among configurations with a given inertia spectrum) and only quasi-periodic relative equilibria in spaces of high enough even dimension are possible. Only the first type is of Kepler type and hence corresponds to the absolute minimum in the classical Sundman inequality.

In a second part, I shall study the relative equilibria of a central configuration in a space of arbitrary dimension and the structure of their bifurcations to quasi-periodic relative equilibria of families of balanced configurations, directing in particular the attention to the angular momenta of such motions,. This question is intimately related to the so-called *Horn problem* which asks for the possible spectra of a sum of hermitian matrices whose spectra are given (see [C2, CJ]).

References

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